

Thursday 18 May 2023 – Afternoon

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

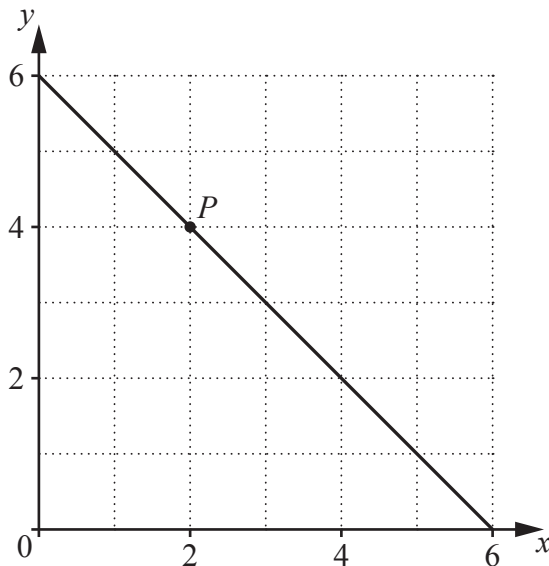
Section A
Pure Mathematics

- 1 (a) Prove that $\cos x + \sin x \tan x \equiv \frac{1}{\cos x}$ (where $x \neq \frac{1}{2}n\pi$ for any odd integer n). [3]
- (b) Solve the equation $2 \sin^2 x = \cos^2 x$ for $0^\circ \leq x \leq 180^\circ$. [2]

- 2 (a) The points A , B and C have position vectors $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 12 \end{pmatrix}$ respectively.
- (i) Show that B lies on AC . [2]
- (ii) Find the ratio $AB : BC$. [1]

- (b) The diagram shows the line $x + y = 6$ and the point $P(2, 4)$ that lies on the line.

A copy of the diagram is given in the Printed Answer Booklet.



The distinct point Q also lies on the line $x + y = 6$ and is such that $|\overrightarrow{OQ}| = |\overrightarrow{OP}|$, where O is the origin.

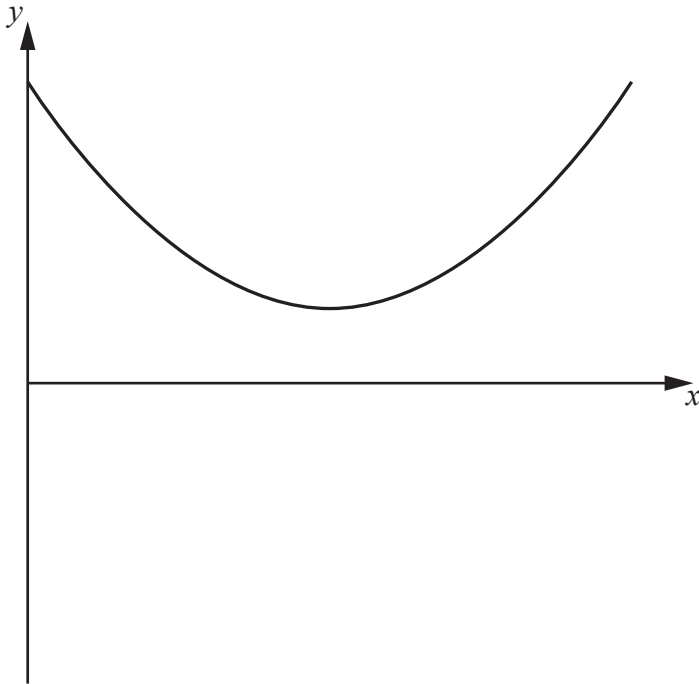
Find the magnitude and direction of the vector \overrightarrow{PQ} . [3]

- (c) The point R is such that \overrightarrow{PR} is perpendicular to \overrightarrow{OP} and $|\overrightarrow{PR}| = \frac{1}{2}|\overrightarrow{OP}|$.

Write down the position vectors of the **two** possible positions of the point R . [2]

- 3 The diagram shows the graph of $y = f(x)$, where $f(x)$ is a **quadratic** function of x .

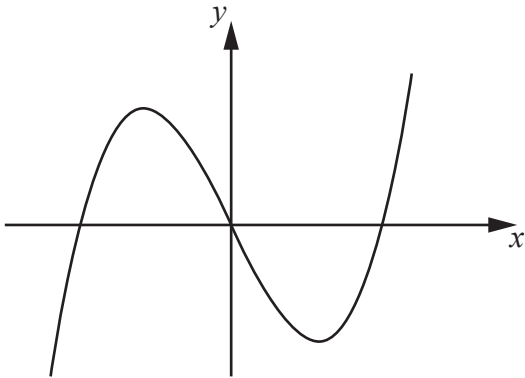
A copy of the diagram is given in the Printed Answer Booklet.



- (a) On the copy of the diagram in the Printed Answer Booklet, draw a possible graph of the gradient function $y = f'(x)$. [3]
- (b) State the gradient of the graph of $y = f''(x)$. [1]
- 4 A curve has equation $y = e^{3x}$.
- (a) Determine the value of x when $y = 10$. [2]
- (b) Determine the gradient of the tangent to the curve at the point where $x = 2$. [2]

5 In this question you must show detailed reasoning.

The diagram shows part of the graph of $y = x^3 - 4x$.



Determine the total area enclosed by the curve and the x -axis. [6]

6 (a) Determine the **two** real roots of the equation $8x^6 + 7x^3 - 1 = 0$. [3]

(b) Determine the coordinates of the stationary points on the curve $y = 8x^7 + \frac{49}{4}x^4 - 7x$. [4]

(c) For each of the stationary points, use the value of $\frac{d^2y}{dx^2}$ to determine whether it is a maximum or a minimum. [4]

7 (a) Two real numbers are denoted by a and b .

(i) Write down expressions for the following.

- The mean of the squares of a and b
- The square of the mean of a and b

[1]

(ii) Prove that the mean of the squares of a and b is greater than or equal to the square of their mean. [3]

(b) You are given that the result in part **(a)(ii)** is true for any two or more real numbers.

Explain what this result shows about the variance of a set of data.

[1]

8 In this question you must show detailed reasoning.

A circle has equation $x^2 + y^2 - 6x - 4y + 12 = 0$. Two tangents to this circle pass through the point $(0, 1)$.

You are given that the scales on the x -axis and the y -axis are the same.

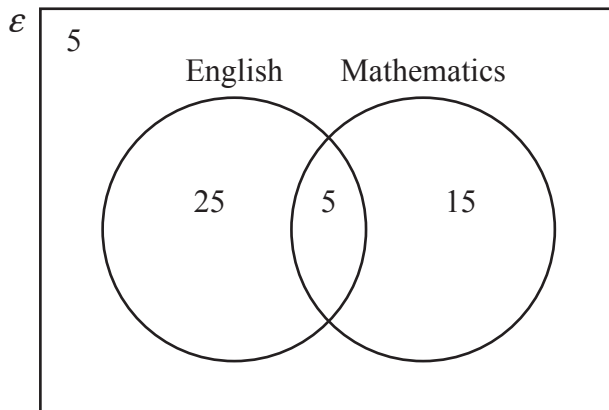
Find the angle between these two tangents.

[7]

Section B
Statistics

- 9 In a survey, 50 people were asked whether they had passed A-level English and whether they had passed A-level Mathematics.

The numbers of people in various categories are shown in the Venn diagram.



- (a) A person is chosen at random from the 50 people.

Find the probability that this person has passed A-level Mathematics.

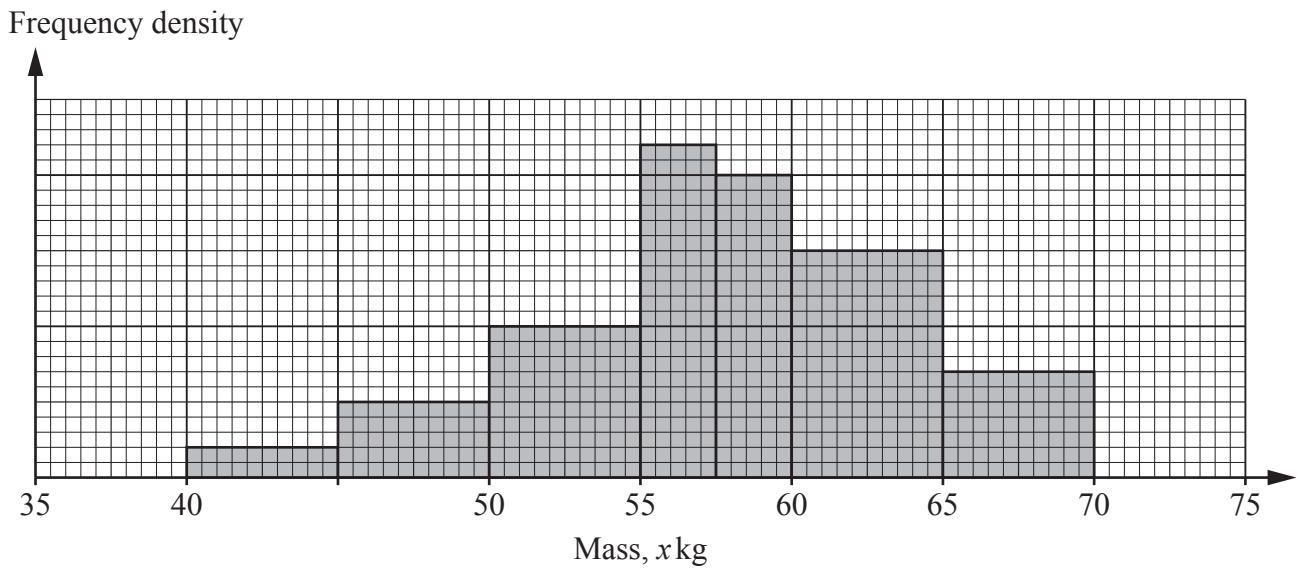
[1]

- (b) Two people are chosen at random, **without replacement**, from those who have passed A-level in at least one of the two subjects.

Determine the probability that both of these people have passed A-level Mathematics.

[3]

- 10 The masses of a random sample of 120 boulders in a certain area were recorded. The results are summarized in the histogram.



- (a) Calculate the number of boulders with masses between 60 and 65 kg. [2]
- (b) (i) Use midpoints to find estimates of the mean and standard deviation of the masses of the boulders in the sample. [3]
- (ii) Explain why your answers are only estimates. [1]
- (c) Use your answers to part (b)(i) to determine an estimate of the number of outliers, if any, in the distribution. [2]
- (d) Give **one** advantage of using a histogram rather than a pie chart in this context. [1]

- 11** Casey and Riley attend a large school. They are discussing the music preferences of the students at their school. Casey believes that the favourite band of 75% of the students is Blue Rocking. Riley believes that the true figure is greater than 75%.

They plan to carry out a hypothesis test at the 5% significance level, using the hypotheses $H_0: p = 0.75$ and $H_1: p > 0.75$.

They choose a random sample of 60 students from the school, and note the number, X , who say that their favourite band is Blue Rocking.

They find that $X = 50$.

- (a)** Assuming the null hypothesis to be true, Riley correctly calculates that $P(X = 50) = 0.0407$, correct to 3 significant figures.

Riley says that, because this value is less than 0.05, the null hypothesis should be rejected.

Explain why this statement is incorrect. [1]

- (b)** Carry out the test. [5]

- (c) (i)** State which mathematical model is used in the calculation in part **(b)**, including the value(s) of any parameter(s). [1]

- (ii)** The random sample was chosen without replacement.

Explain whether this invalidates the model used in part **(b)**. [1]

12 This question deals with information about the populations of Local Authorities (LAs) in the North of England, taken from the 2011 census.

Fig. 1

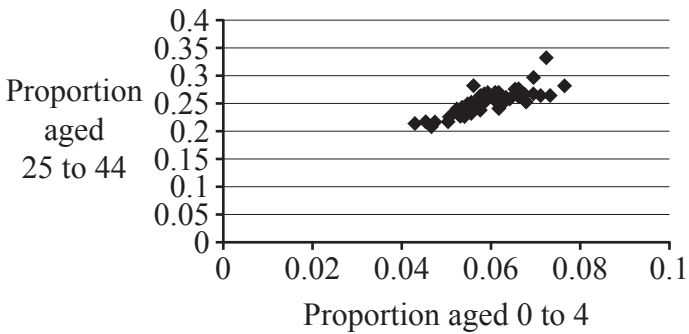


Fig. 2

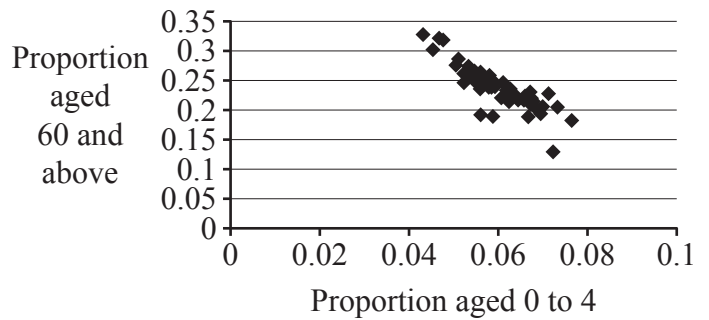


Fig. 1 and Fig. 2 both show strong correlation, but of two different kinds.

- (a) For each diagram, use a single word to describe the kind of correlation shown. [1]
- (b) For each diagram, suggest a reason, in context, why the correlation is of the particular kind described in part (a). [2]

Fig. 3 is the same as Fig. 2 but with the point A marked.

Fig. 4 shows information about the same LAs as Fig. 2 and Fig. 3.

Fig. 3

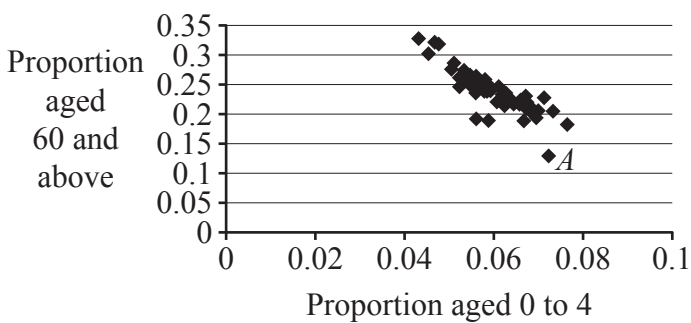
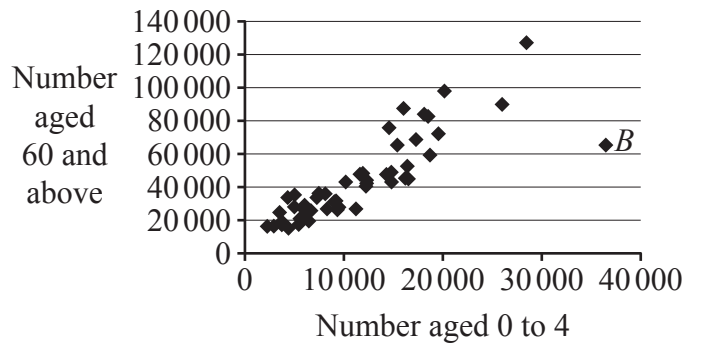


Fig. 4



- (c) Point A in Fig. 3 and point B in Fig. 4 represent the same LA.

Explain how you can tell that this LA has a large population. [1]

END OF QUESTION PAPER

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